MATH 1101: An Introduction to Optimization

Fall 2025 Course Syllabus

- **Lecture**
  - Section 1: Tuesday 13:50 – 16:25 at S202
  - Section 2: Thursday 13:50 – 16:25 at S202

- **Instructor:** Guanqiang "Tim" Zhou <guanqiang.zhou@scupi.cn>

- **Office Hours:** Tuesday & Wednesday & Thursday 10:30 – 13:30 at N512 (or make an appointment via email)

- **Teaching Assistants**
  - Section 1: Yongyuan (Derrick) Hu <2023141520029@stu.scu.edu.cn>
  - Section 2: Peiyang (Hank) Zou <1374186059@qq.com>

- **Main Reference:** *Convex Optimization*, by Stephen Boyd and Lieven Vandenberghe, Cambridge University Press. Free download available at stanford.edu. Supplementary readings will be posted on Blackboard.

Course Description

This course provides a rigorous introduction to optimization, focusing on both the theory and algorithms that underlie modern applications in engineering, data science, and operations research. Beginning with fundamental mathematical concepts, students will learn how to formulate optimization problems, analyze their structure, and apply theoretical tools such as duality and optimality conditions. The course also introduces key algorithmic methods, emphasizing convergence, complexity, and practical implementation.

Course Objectives

By the end of this course, students will be able to:

1. Recognize and formulate optimization problems in diverse domains.
2. Understand the mathematical foundations of optimization, including convex sets, convex functions, and related function properties.
3. Apply optimality conditions (first-order, second-order, KKT) to analyze problems.
4. Use duality theory to interpret and solve optimization problems.
5. Implement and analyze basic optimization algorithms, including gradient descent, Newton's method, and interior-point methods.
6. Appreciate the applications of optimization in engineering, data science, and beyond.

Learning Outcomes

Upon successful completion, students will be able to:

- Formulate real-world problems as mathematical optimization problems.
- Classify problems as convex, nonconvex, linear, or nonlinear.
- Apply convex analysis tools to study problem feasibility and structure.
- Derive and interpret optimality conditions.
- Use duality theory to analyze sensitivity and design algorithms.
- Implement and analyze gradient-based and Newton-type optimization algorithms.
- Assess trade-offs between accuracy, computational complexity, and convergence rate.

Tentative Lecture Schedule

- **Week 1:** Introduction to Optimization — applications, terminology, standard form, motivating examples.
- **Weeks 2–4:** Fundamental Concepts — convex sets, affine sets, cones, polyhedra; convex functions and properties (monotonicity, Lipschitz continuity, smoothness); mathematical tools (gradients, Hessians, level sets, norms).

- **Weeks 5–7:** Optimality and Duality Theory — optimality conditions for unconstrained and constrained problems; Lagrangian, dual functions, weak/strong duality; sensitivity analysis, complementary slackness; Karush-Kuhn-Tucker (KKT) conditions.
- **Week 8:** Midterm Exam.
- **Weeks 9–12:** First-Order Algorithms — gradient descent and subgradient methods; convergence analysis of first-order methods; accelerated gradient methods (Nesterov); projected gradient methods for constrained problems.
- **Weeks 13–14:** Second-Order Algorithms — Newton's method and convergence properties; quasi-Newton methods; barrier and interior-point methods (overview).
- **Week 15:** Algorithm Complexity & Advanced Topics — complexity of convex optimization; trade-offs between first-order and second-order methods; connections to machine learning and large-scale optimization.
- **Week 16:** Review & Applications — applications in signal processing, control, and machine learning; student-selected examples and case studies.
- **Week 17:** Final Exam.

Grading

- Attendance and Participation: 10%
- Homework Assignments: 20%
- Midterm Exam: 30%
- Final Exam: 40%

Code of Academic Conduct

Students are expected to adhere to the highest standards of academic integrity.

- **Collaboration**: Discussion of concepts and general approaches is encouraged, but all submitted work must be your own.

- **Plagiarism**: Copying from peers, prior semesters, online sources, or AI-generated content without attribution is strictly prohibited.
- **Exams**: Closed-book unless otherwise specified. Use of unauthorized materials will result in disciplinary action.
- **Citations**: Any external sources used in assignments or presentations must be properly cited.
- **Consequences**: Violations will be handled in accordance with university policy and may result in penalties ranging from assignment failure to course failure or referral to the academic integrity office.