Technical Elective: Tensor Analysis in Materials
Fall 2024

Instructor

Dr. Liwei Geng
South campus Rm 402
Email: liwei.geng@scupi.cn

Meeting Time & Location

Tuesday 8:15-11:00 at RM 4-201

Office Hour

Wednesday & Thursday: 13:00-17:00, or by appointment

TA information

Feiyang Liu: 2021141520087@stu.scu.edu.cn
QQ group: 367857384

Course Materials

Textbook

- J. F. Nye, "Physical Properties of Crystals: Their Representation by Tensors and Matrices", Oxford University Press (1985)

Class notes

Handouts

Reference Textbooks

- Robert E. Newnham, "Properties of materials: anisotropy, symmetry, structure", Oxford University Press (2005)
- W. Michael Lai, David Rubin, Erhard Krempl, "Introduction to Continuum Mechanics", 3rd edition, Butterworth-Heinemann Ltd., Boston (1993)

Course Description

A comprehensive treatment of deformation-related physical behaviors of materials in the mathematical framework of tensor analysis. Development of elastic constitutive relations. Introduction to non-elastic strain associated with thermal expansion, ferroelectricity, and magnetism. Relations between material symmetry and tensor properties.

Course Objectives

Upon successful completion of this course, the students will be able to:

1. Utilize tensor algebra and calculus of tensor fields;
2. Understand stress, strain, and the constitutive relations;
3. Determine the effects of material symmetry on the physical properties;
4. Analyze the internal stress due to thermal expansion, electrostriction and magnetostriction.

ABET Learning Outcomes:

- An ability to identify, formulate, and solve complex engineering problems by applying principles of engineering, science, and mathematics.

- An ability to develop and conduct appropriate experimentation, analyze and interpret data, and use engineering judgment to draw conclusions.
- An ability to acquire and apply new knowledge as needed, using appropriate learning strategies.

Prerequisite

MATH 0280: Matrices and Linear Algebra
ENGR 0022: Materials Structures and Properties

Grading

Homework	40%
Participation	5%
Project report	15%
Final Exam	40%

Course Policies

1. Show up on time.
2. It is OK to discuss homework assignments with your classmates, but all submissions must be your own work.
3. It is expected that you will work on assignments consistently from the day they are made available.

Late Assignment Policy

10% deduction/day

Grade Policy

A: 90 – 100	A−: 85 – 89	B+: 80 – 84	B: 76 – 79	B−: 73 – 75
C+: 70 – 72	C: 66 – 69	C−: 63 – 65	D: 60 – 62	F: < 60

Evaluation Policy

Partial credit will be awarded to recognize that some portion of the work is correct. However, partial credit grading is only practical if the work is clearly developed, with clear and well-marked diagrams when fitting, with the appropriate equations prominently displayed, where the substitutions into the equations are quite clear, and the assumptions used are obvious to the grader. That is, it is the student's responsibility to present her/his work so clearly that the grader can quickly ascertain the location and nature of the error(s) and can follow the subsequent work through. If this is not clear on the work submitted, credit cannot be given (then or later). ***Partial credit is assigned at the discretion of the grader.*** It is therefore always in your best interest to practice clarity and completeness in your solutions when working homework problems. This is applicable to exam problems as well.

Copyrights

The handouts used in this course are copyrighted. By "handouts" we mean all materials generated for this class, which include but are not limited to syllabi, in-class materials, videos, review sheets, and additional problem sets. Because these materials are copyrighted, you do not have the right to copy or distribute the handouts, unless the author expressly grants permission.

Academic Integrity

All students are expected to adhere to the standards of academic honesty. Any student engaged in cheating, plagiarism, or other acts of academic dishonesty would be subject to disciplinary action. Any student suspected of violating this obligation for any reason during the semester will be required to participate in the procedural process, initiated at the instructor level, as outlined in the University Guidelines on Academic Integrity. This may include but is not limited to the confiscation of the examination of any individual suspected of violating the University Policy.

Course Contents

<u>Chapter 1 General Principles</u>

1.1 Indicial notation
1.2 Vector
1.3 Tensor (second-rank)
1.4 Transformation laws
1.5 Eigenvalues and eigenvectors
1.6 Mohr circle
1.7 Examples of second-rank tensor
1.8 Material symmetry
1.9 Gradient, divergence, curl, and Gaussian theorem

<u>Chapter 2 Stress</u>

2.1 Force
2.2 Stress tensor
2.3 Conservation of linear momentum & moment of momentum
2.4 Proof that σ_{ij} form a tensor
2.5 Normal stress and shear stress
2.6 Mohr circle
2.7 Special forms of the stress tensor
2.8 Difference between the stress tensor and tensors representing crystal properties

<u>Chapter 3 Strain</u>

3.1 Description of motion
3.2 Deformation gradient
3.3 Lagrangian strain
3.4 Infinitesimal strain

<u>Chapter 4 Elasticity</u>

4.1 Hooke's law
4.2 Matrix notation/Voigt notion/contracted form
4.3 The effect of material symmetry
4.4 Young's modulus
4.5 Shear modulus, bulk modulus, and Poisson's ratio
4.6 Elastic properties of polycrystals
4.7 Stress-strain relation for an isotropic material
4.8 Navier equations

<u>Chapter 5 Stress-free strain (spontaneous strain/eigenstrain)</u>

5.1 Introduction
5.2 Thermal expansion

5.3 Strain mismatch at the “stress-free strain domain” boundaries (internal stress)
5.4 Electrostriction
5.5 Magnetostriction

Project

<u>Requirement:</u> Written Report

<u>Options of topics:</u>

Select an unexplored tensor quantity that has not been covered in this course and conduct a comprehensive analysis. Suitable tensor quantities encompass, but are not restricted to, the piezoelectric coefficient (d_{ij}), electrical/magnetic susceptibility (χ_{ij}), voltage coefficient (g_{ij}), electromechanical coupling coefficient (k_{ij}), Poisson's ratio (ν_{ij}), electrostrictive coefficient (Q_{ij}), magnetostriction coefficient (λ_{ij}), expansion coefficient (α_{ij}), etc. (For the designated tensor, kindly incorporate the following details in your report: name, definition, proof of tensor nature, matrix representation under crystal symmetry, mathematical relationships with other relevant quantities, typical values in various materials.)

Table 1.2. Materials property and transport tensors (adapted from Nowick (Nowick, 1995)).

Property	Symbol	Field	Response	Type/#
	Tensors of Rank 0 (Scalars)			
Specific heat	C	ΔT	$T\Delta S$	E/1
	Tensors of Rank 1 (Vectors)			
Electrocaloric	p_i	E_i	ΔS	E/3
Magnetocaloric	q_i	H_i	ΔS	E/3
Pyroelectric	p'_i	ΔT	D_i	E/3
Pyromagnetic	q'_i	ΔT	B_i	E/3
	Tensors of Rank 2			
Thermal expansion	α_{ij}	ΔT	ϵ_{ij}	E/6
Piezocaloric effect	α'_{ij}	σ_{ij}	ΔS	E/6
Dielectric permittivity	κ_{ij}	E_j	D_i	E/6
Magnetic permeability	μ_{ij}	H_j	B_i	E/6
Optical activity	g_{ij}	$l_i l_j$	G	E/6
Magnetoelectric polarization	λ_{ij}	H_j	D_i	E/9
Converse magnetoelectric polarization	λ'_{ij}	E_j	B_i	E/9
Electrical conductivity (resistivity)	σ_{ij} (ρ_{ij})	E_j (j_j)	j_i (E_i)	T/6
Thermal conductivity	K_{ij}	$\nabla_j T$	h_i	T/6
Diffusivity	D_{ij}	$\nabla_j c$	m_i	T/6
Thermoelectric power	Σ_{ij}	$\nabla_j T$	E_i	T/9
Hall effect	R_{ij}	B_j	ρ_i^a	T/9
	Tensors of Rank 3			
Piezoelectricity	d_{ijk}	σ_{jk}	D_i	E/18
Converse piezoelectricity	d'_{ijk}	E_k	ϵ_{ij}	E/18
Piezomagnetism	Q_{ijk}	σ_{jk}	B_i	E/18
Converse piezomagnetism	Q'_{ijk}	H_k	ϵ_{ij}	E/18
Electro-optic effect	r_{ijk}	E_k	$\Delta\beta_{ij}$	E/18
Nernst tensor	Σ_{ijk}	$\nabla_j T B_k$	E_i	T/27
	Tensors of Rank 4			
Elasticity	s_{ijkl} (c_{ijkl})	σ_{kl} (ϵ_{kl})	ϵ_{ij} (σ_{ij})	E/21
Electrostriction	γ_{ijkl}	$E_k E_l$	ϵ_{ij}	E/36
Photoelasticity	q_{ijkl}	σ_{kl}	$\Delta\beta_{ij}$	E/36
Kerr effect	p_{ijkl}	$E_k E_l$	$\Delta\beta_{ij}$	E/36
Magnetoresistance	ξ_{ijkl}	$B_k B_l$	ρ_{ij}^s	T/36
Piezoresistance	Π_{ijkl}	σ_{kl}	$\Delta\rho_{ij}$	T/36
Magnetothermoelectric power	Σ_{ijkl}	$\nabla_j T B_k B_l$	E_i	T/54
Second order Hall effect	ρ_{ijkl}	$B_j B_k B_l$	ρ_i^2	T/30
	Tensors of Rank 6			
Third order elasticity	c_{ijklmn}	$\epsilon_{kl}\epsilon_{mn}$	σ_{ij}	E/56