# Differential Equations - MATH 0290 Fall 2023 – 24



# **Course Description**

Instructor

Mushtaq Loan <u>mushtaq.loan@scupi.cn</u> Office: 3-324B SCUPI Building

Classroom:3 – 201 SCUPI BuildingLectures:Section 1 – Monday, 1:50 – 4:30<br/>Section 2 – Tuesday, 8.15 – 11:00Office Hours:Wednesday, 2 - 3 PM, Friday 2 – 3 PM

Teaching Assistants: Mike Gao,<br/>Shao CiliangEmail - 2021141520094@stu.scu.edu.cnShao Ciliang2021141520227@stu.scu.edu.cn

Tutorials: TBA

# **Course Description**

The laws of nature are expressed as differential equations. Most "real life" systems that are described mathematically, be they physical, biological, financial, or economic, are described by means of differential equations. Scientists and engineers must know how to model the world in terms of differential equations, and how to solve those equations and interpret the solutions. Our ability to predict the way in which these systems evolve or behave is determined by our ability to model these systems and find solutions of the equations explicitly or approximately. This course focuses on linear differential equations and their applications in engineering. Topics include the solution of first- and higher order differential equations, power series solutions, interpretation of solutions, Laplace transforms, partial differential equations, and applications. The course emphasizes classical methods for finding exact solution formulas. We will also study numerical methods which yield precise but approximate quantitative information, and qualitative methods which provide a rich geometric understanding of ODE. Rather than viewing them as isolated topics, students leave the course with an enhanced appreciation for the connections between differential equations and fields of engineering. This course prepares the students for success in disciplines which rely on differential equations, and in more advanced mathematics which incorporate these topics.

# Prerequisites

MATH 0220 & 0230 - Analytic Geometry and Calculus 1 & 2



# **Course Learning Objectives (CLOs)**

- CLO1 Evaluate first order differential equations including separable, homogeneous exact, and linear. Show existence and uniqueness of solutions.
- CLO2 Perform both exact and numerical procedures for finding solutions to problems of differential equations, interpret and present graphical and qualitative representations of solutions to problems.
- CLO3 Students understand fundamental concepts of differential equations, and the interrelationship between differential equations and linear algebra.
- CLO4 Create and analyse mathematical models using higher order differential equations to solve application problems in ECE, ME, CE.

# Student Learning Outcomes (SLOs)

After the completion of the course, Students will be able to:

- SLO1 model a physical system to obtain a first and second order differential equations.
- SLO2 solve first and higher order differential equations utilizing the standard techniques for separable, exact, linear, homogeneous, or Bernoulli cases.
- SLO3 find the complete solution of a nonhomogeneous differential equation as a linear combination of the complementary function and a particular solution.
- SLO4 solve the main equations with various input functions including zero, constants, exponentials, sinusoids, step functions, impulses, and superpositions of these functions.
- SLO5 test the plausibility of a solution to a differential equation (DE) which models a physical situation by using reality-check methods such as physical reasoning, looking at the graph of the solution, testing extreme cases, and checking units.
- SLO6 visualize solutions using direction fields and approximate them using Euler's method.
- SLO7 derive a basic first-order ODE model from a description of a physical system. Understand and apply basic numerical methods for solving ODEs

#### Resources

#### Textbook:

*Differential Equations with Boundary Value Problems, Second Edition,* John Polking, Albert Boggess, David Arnold, Pearson Education, Inc. 2010, NJ, USA.

#### Supplementary:

#### Differential Equations and Boundary Value Problems, Computing and Modeling, Fifth

Edition, C. Henry Edwards, David E. Penney, and David Calvis, Pearson Education, Inc. 2015, NJ, USA.

#### **Computational Resources**

Students will use ANSYS and MATLAB as effective tools for real-world processes or systems operations.

A range of software is available to current students including MATLAB and ANSYS.



#### Blackboard

The Learning Management System provides easy access to your online services such as Blackboard learning environment (<u>https://pibb.scu.edu.cn/</u>). The SCUPI online learning environment hosts lecture notes, class discussions, module resources and electronic resources. The Blackboard support team can also assist with Turnitin queries. Lecture notes, online quizzes, assignments, projects, announcements, and your grades will be uploaded on the MATH 0290 page of the Blackboard.

# **Course Format**

The course has been organized to follow the sequence of topics covered in the textbook on Differential Equations. There are four major units.

- First Order Differential Equations
- Second Order Constant Coefficient Linear Equations
- Laplace Transform and Fourier Series
- First Order Systems and Partial Differential Equations

Each unit is divided into sessions, which consist of written notes, lectures, lecture notes, tutorials, practice problems, and problem sets. Following the practice at SCUPI, the problem sets are split into two parts: Part I covering simple problems designed to emphasize a specific skill or technique, and Part II covering harder, often multistep problems, designed to help the student learn to apply the skills and techniques to more realistic problems. Complete solutions are provided for all problem sets.

At the end of each unit is an assignment covering the material in the unit and a practice exam to help you prepare for the exam. Solutions are included for practice exam.

At the end of Unit IV is a final exam covering the entire course.

#### Assignments

There will be 4 application assignments during the course. Each assignment presents a connection to the real-world problems in different engineering majors. Besides the greatly increased learning intensity that comes from personal involvement with a project, and the chance to show graduate schools and potential employers the student's ability to initiate and carry out a complex task, it gives the student an introduction to mathematics as it is: a living and developing intellectual organism where progress is achieved by the interplay of individual creativity and collective knowledge.

Essay is typed with appropriate font and margins. Title page includes a specific essay title, your name, and school affiliation. Each page includes a page number and a running head (in the header – not typed separately on each page). Outside sources are properly cited when used. Length is as specified in assignment description. No late submission is accepted, and plagiarism is not tolerated. Discussions of the assignment problems are encouraged, but each student must submit his/her assignment. A grading rubric is used for objective and consistent

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assessment of various components of the assignment. The rubrics for the assignments will be uploaded to the Blackboard. SCUPI expects its students to spend an average of 3 or more hours working through each of the 18 sessions.

**Online Quizzes**: Online quizzes are assigned every second week. There are 15 online quizzes and best of 10 will be considered for the final grade.

## **Course Assessment**

The final grade will be computed based on the score of weekly assignments, quizzes, research project, and final exam. The final exam is comprehensive and closed-book.

## Final Grade

The final grade will be computed according to the following scheme:Homework & Attendance10%Assignments:30%Quizzes:10%Midterm:20%Final Exam:30%

## Cutoffs

A [90, 100], A- [85, 90), B+ [80, 85), B [76, 80), B- [73, 76), C+ [70, 73), C [66, 70), C- [63, 66), D+ [61, 63), D [60, 61), F (60, 0).



# **Course Outline**

# Schedule and weekly learning goals

The schedule is tentative and subject to change. The listed objects below should be viewed as the key concepts you should grasp after each week and also as a study guide before each exam and at the end of the semester. Each test will be based on material taught up until the second last week before the test; namely, Test 1 covers Weeks 02-08, and Test 2 is based on Weeks 09-13. The final exam will cover all topics taught in this semester.

Week	Dates	Chapter/Section	Topics
01	26/02 – 01/03	2.1-2.4 First-order Differential Equations	<ul> <li>Linear Equations</li> <li>Mixing Problems</li> <li>Exact Differential Equations</li> </ul>
02	05/03 – 09/03	2.5 – 2.9 First-order Differential Equations	<ul> <li>Existence and Uniqueness of Solutions</li> <li>Dependence of Solutions on Initial Conditions</li> <li>Autonomous Equations and Stability Electrical Circuits</li> </ul>
03	11/03 – 15/03	3.1 – 3.5 Modelling and Applications	<ul> <li>Modelling Population Growth</li> <li>Models and the Real World</li> <li>Model for Hot Water Heater</li> <li>Heat Conduction through a Wall</li> <li>Electrical Circuits</li> </ul>
		Assignment 1	First-order Differential Equations
04	18/03 – 22/03	4.1 – 4.4 Second-order Differential Equations	<ul> <li>Second-order Equations and Systems</li> <li>Linear, Homogeneous Equations with Constant Coefficients</li> <li>Harmonic Motion</li> </ul>
05	25/03 – 29/03	4.5 – 4.6 Second-order Differential Equations	<ul> <li>Inhomogeneous Equations; the Method of Undetermined Coefficients</li> <li>Variation of Parameters</li> <li>Forced Oscillations and Resonance</li> </ul>
06	01/04 - 05/04	4.7 Second-order Equations (Applications)	<ul> <li>Electrical Circuits</li> <li>Radial Heat Conduction</li> <li>Endpoint Problems and Eigenvalues</li> </ul>
		Assignment 2	Second-order Differential
07	00/04 40/05	54 52	Equations
07	08/04 - 12/04	5.1 – 5.3 Laplace Transform Methods	<ul> <li>The Definition of the Laplace Transform</li> <li>Basic Properties of the Laplace Transform</li> <li>The Inverse Laplace Transform</li> </ul>

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11	15/04 - 19/04	5.4 – 5.7	<ul> <li>Using the Laplace Transform to Solve</li> </ul>
		Laplace	Differential Equations
		Methods	<ul> <li>Discontinuous Forcing Terms</li> </ul>
		Wiethous	The Delta Function
			<ul> <li>Forced Oscillations Revisited</li> </ul>
09	22/04 - 26/04	6.1 - 6.4	<ul> <li>Euler's Method</li> </ul>
		Numerical	<ul> <li>Runge-Kutta Methods</li> </ul>
		wiethous	<ul> <li>Numerical Error Comparisons</li> </ul>
			<ul> <li>Practical Use of Solvers</li> </ul>
	22/04 - 26/04	Midterm	• Chap 2 - 8
10	22/04 – 26/04	8.1 - 8.5	<ul> <li>Geometric Interpretation of Solutions</li> </ul>
		Linear Systems	<ul> <li>Qualitative Analysis</li> </ul>
			Linear Systems
			<ul> <li>Properties of Linear Systems</li> </ul>
11	29/04 - 03/05	9.1-9.4	Planar Systems
		Linear System	<ul> <li>Phase Plane Portraits</li> </ul>
		With Constant	<ul> <li>The Trace-Determinant Plane</li> </ul>
		coements	<ul> <li>Higher-Dimensional Systems</li> </ul>
12	06/05 – 10/05	9.5 – 9.9	<ul> <li>The Exponential of a Matrix</li> </ul>
		Linear System	<ul> <li>Qualitative Analysis of Linear Systems</li> </ul>
		with Constant	<ul> <li>Higher-Order Linear Equations</li> </ul>
		Coements	<ul> <li>Inhomogeneous Linear Systems</li> </ul>
		Assignment 3	Numerical Methods
		Assignments	
13	13/05 – 17/05	12.1 – 12.3	<ul> <li>Periodic Functions and Trigonometric</li> </ul>
13	13/05 – 17/05	12.1 – 12.3 Fourier Series	<ul> <li>Periodic Functions and Trigonometric Series</li> </ul>
13	13/05 – 17/05	12.1 – 12.3 Fourier Series	<ul> <li>Periodic Functions and Trigonometric Series</li> <li>General Fourier Series and Convergence</li> </ul>
13	13/05 – 17/05	12.1 – 12.3 Fourier Series	<ul> <li>Periodic Functions and Trigonometric Series</li> <li>General Fourier Series and Convergence</li> <li>Fourier Sine and Cosine Series</li> </ul>
13	13/05 – 17/05 20/05 – 24/05	12.1 – 12.3 Fourier Series	<ul> <li>Periodic Functions and Trigonometric Series</li> <li>General Fourier Series and Convergence</li> <li>Fourier Sine and Cosine Series</li> <li>Applications of Fourier Series</li> </ul>
13	13/05 - 17/05 20/05 - 24/05	12.1 – 12.3 Fourier Series 12.4 – 12.5 Fourier Series	<ul> <li>Periodic Functions and Trigonometric Series</li> <li>General Fourier Series and Convergence</li> <li>Fourier Sine and Cosine Series</li> <li>Applications of Fourier Series</li> <li>The Discrete Fourier Transform and the</li> </ul>
13	13/05 – 17/05 20/05 – 24/05	12.1 – 12.3 Fourier Series 12.4 – 12.5 Fourier Series	<ul> <li>Periodic Functions and Trigonometric Series</li> <li>General Fourier Series and Convergence</li> <li>Fourier Sine and Cosine Series</li> <li>Applications of Fourier Series</li> <li>The Discrete Fourier Transform and the FFT</li> </ul>
13	13/05 - 17/05 20/05 - 24/05	12.1 – 12.3 Fourier Series 12.4 – 12.5 Fourier Series Assignment 4	<ul> <li>Periodic Functions and Trigonometric Series</li> <li>General Fourier Series and Convergence</li> <li>Fourier Sine and Cosine Series</li> <li>Applications of Fourier Series</li> <li>The Discrete Fourier Transform and the FFT</li> <li>Partial Differential Equations</li> </ul>
13 14 15	13/05 - 17/05 20/05 - 24/05 27/05 - 31/05	12.1 – 12.3 Fourier Series 12.4 – 12.5 Fourier Series Assignment 4 13.1 – 13.2	<ul> <li>Periodic Functions and Trigonometric Series</li> <li>General Fourier Series and Convergence</li> <li>Fourier Sine and Cosine Series</li> <li>Applications of Fourier Series</li> <li>The Discrete Fourier Transform and the FFT</li> <li>Partial Differential Equations</li> <li>Derivation of the Heat Equation</li> </ul>
13 14 15	13/05 - 17/05 20/05 - 24/05 27/05 - 31/05	12.1 – 12.3 Fourier Series 12.4 – 12.5 Fourier Series Assignment 4 13.1 – 13.2 Partial	<ul> <li>Periodic Functions and Trigonometric Series</li> <li>General Fourier Series and Convergence</li> <li>Fourier Sine and Cosine Series</li> <li>Applications of Fourier Series</li> <li>The Discrete Fourier Transform and the FFT</li> <li>Partial Differential Equations</li> <li>Derivation of the Heat Equation</li> <li>Separation of Variables for the Heat</li> </ul>
13 14 15	13/05 - 17/05 20/05 - 24/05 27/05 - 31/05	12.1 – 12.3 Fourier Series 12.4 – 12.5 Fourier Series Assignment 4 13.1 – 13.2 Partial Differential Equations	<ul> <li>Periodic Functions and Trigonometric Series</li> <li>General Fourier Series and Convergence</li> <li>Fourier Sine and Cosine Series</li> <li>Applications of Fourier Series</li> <li>The Discrete Fourier Transform and the FFT</li> <li>Partial Differential Equations</li> <li>Derivation of the Heat Equation</li> <li>Separation of Variables for the Heat Equation</li> </ul>
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13 14 15 16	13/05 - 17/05 20/05 - 24/05 27/05 - 31/05 03/06 - 07/06	12.1 – 12.3 Fourier Series 12.4 – 12.5 Fourier Series Assignment 4 13.1 – 13.2 Partial Differential Equations	<ul> <li>Periodic Functions and Trigonometric Series</li> <li>General Fourier Series and Convergence</li> <li>Fourier Sine and Cosine Series</li> <li>Applications of Fourier Series</li> <li>The Discrete Fourier Transform and the FFT</li> <li>Partial Differential Equations</li> <li>Derivation of the Heat Equation</li> <li>Separation of Variables for the Heat Equation</li> <li>The Wave Equation</li> <li>Laplace's Equation</li> </ul>
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13 14 14 15 16 16	13/05 - 17/05 20/05 - 24/05 27/05 - 31/05 03/06 - 07/06 10/06 - 14/06	12.1 - 12.3 Fourier Series $12.4 - 12.5$ Fourier Series $Assignment 4$ $13.1 - 13.2$ Partial Differential Equations $13.3 - 13.4$ $13.5$	<ul> <li>Periodic Functions and Trigonometric Series</li> <li>General Fourier Series and Convergence</li> <li>Fourier Sine and Cosine Series</li> <li>Applications of Fourier Series</li> <li>The Discrete Fourier Transform and the FFT</li> <li>Partial Differential Equations</li> <li>Derivation of the Heat Equation</li> <li>Separation of Variables for the Heat Equation</li> <li>The Wave Equation</li> <li>Laplace's Equation</li> <li>Steady-State Temperature and Laplace's Equation</li> <li>The Heat and Wave Equations in Higher Dimensions</li> <li>The Heat and Wave Equations in Higher</li> </ul>
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# **Course Policies**

#### **Attendance Policy**

Attendance is expected in all lectures. Valid excuses for absence will be accepted before class. In extenuating circumstances, valid excuses with proof will be accepted after class.

# Policies on Late Assignments and Exams

Students should start their homework assignments immediately after the assignments are given, and DO NOT wait until the last minute to meet the deadlines. Late assignments will be NOT accepted except for emergencies and health issues. Any other late assignments handed in will be marked but will be given a zero mark. At most, two extensions for assignments will be given in this course. All assignments will be counted in your total grade. Late submission for previous assignments during the final exam period will NOT be accepted in any form for any excuses.

All tests and the final exam are mandatory. There will be no makeup exam for each test. If you miss the final, a makeup exam may be given for the final exam if the student has approval from the instructor or emergencies and health issues with valid proof. I will not accept the student deceleration for absence form for the final exam.

## Academic Assistance

If you have questions regarding class materials, homework problems, grading issues, etc., you are encouraged to attend office hours. Unless you cannot make it, you may email the TA or the instructor. Please allow 24 to 48 hours for any response to emails. The subject of each email must include "[MATH0290]". For example, if you have a question regarding a homework problem, the email's subject could be [MATH0290] Question about Problem X of Assignment X. Please make sure that you sign off with your official name (the one that appears in Blackboard). You are encouraged to use academic language in your posts.

#### Academic Integrity

At Sichuan University, we are guided in all of our work by the values of academic integrity: honesty, trust, fairness, responsibility, and respect. As a student, you are required to demonstrate these values in all your work. Everyone at SCUPI is expected to treat others with dignity and respect. The Code of Student Conduct allows Sichuan University to take disciplinary action if students do not follow this community expectation.